Pergamon

J. Appl. Maths Mechs, Vol. 58, No. 1, pp. 73–80, 1994

0021–8928/94 $24.00+0.00

0021–8928(94)E0025-6

MATHEMATICAL MODELLING OF THE ACTION OF VIBRATIONS ON A GAS-SATURATED COAL SEAM†

V. V. LYAKH and S. P. MINEYEV

Dnepropetrovsk

(*Received* 25 *September* 1992)

The problem of the propagation of vibration waves in an elastic coal-bearing rock is considered based on a model of a three-component heterogeneous medium. The effect of the parameters of the vibration on the weakening of the rock mass and on the free and absorbed gas distribution in the vicinity of a vibration well is estimated.

Up to now the changes in stress distribution in a rock mass under the action of a vibration wave have been considered within the framework of the linear theory of elasticity and a two-component medium represented by an elastic matrix and a free gas [1, 2]. In order to establish the optimum action of the vibrations it is crucial that the mathematical model be close to a real coal seam, which is a medium with pores and cracks saturated by a two-phase, i.e. free and absorbed, gas. In this connection a new boundary-value problem of vibration-wave propagation in a coal seam is stated. It is based on the mathematical model of a three-component heterogeneous medium developed by Podil'chuk [2, 3], which describes the behaviour of a gas-saturated rock mass.

1. PHYSICAL BASIS OF THE MODEL

Following the ideas of [2], when considering a primary macrovolume of rock mass we will distinguish three basic components: a deformable matrix with cracks and pores, as well as free and absorbed methane. In a perturbed rock mass the free gas moves from the cracks and macropores into the excavations or wells, basically obeying the laminar filtration described by Darcy's law

$$m_2 V_i^{12} = -k\mu_1^{-1} \partial P_2 / \partial x_i, \quad i = 1, 2, 3 \tag{1.1}$$

Here m_2 is the matrix porosity, equal to the relative volume of macropores and cracks, V_i^{12} are the components of the mean velocity vector of the free gas relative to the solid component, k is the gas permeability coefficient μ_1 is the viscosity of the gas, P_2 is the gas pressure in the macropores and cracks, and κ_i are the macrocoordinates of averaged motion. The state of the free gas is described by the equation

$$P_2 = \rho_2 ZRT \tag{1.2}$$

†*Prikl. Mat. Mekh.* Vol. 58, No. 1, pp. 69–76, 1994.

Here ρ_2 is the mean density of the gas in the macropores, Z is the gas compressibility coefficient, R is the universal gas constant, and T is the absolute temperature.

At the same time as the free gas filters through the coal, the absorbed gas undergoes diffusion inside the system of macropores. The diffusion flow is described by Fick's law

$$\rho_3 m_3 V_i^{13} = -D\partial(c_3 + a) / \partial x_i, \quad i = 1,\ 2,\ 3 \tag{1.3}$$

(m_3 is the volume ratio of macropores in the medium, ρ_3 is the mean density of the gas in the macropores, V_i^{13} are the components of the mean velocity vector of gas motion within the system of macropores relative to the solid component, D is the diffusion coefficient, and c_3 and a are the concentration of the free gas and the quantity of absorbed gas in the micropores per unit volume of the medium).

The filtration and diffusion fluxes are related by the gas transfer from the micropores into the macropores and cracks due to the difference in pressure and concentration of the gas. The dependence of the gas transfer on the gas concentrations c_2 and c_3 in the micro- and macropores is given by

$$q = \beta(c_2 - c_3) \tag{1.4}$$

where β is the gas transfer coefficient.

In an unperturbed rock mass the amount of free gas is much smaller than the amount of absorbed gas [4]. It follows that the amount a of absorbed gas is much greater than the free gas concentration c_3 in the micropores. This enables us to simplify the model by neglecting the free gas concentration in micropores and henceforth setting $c_3 = 0$. Taking this into account, we can write the gas density inside the micropores in the form

$$m_3 \rho_3 = a \tag{1.5}$$

It is obvious that $c_2 = \rho_2$. Then the gas transfer from the system of micropores into the macropores and cracks can be determined from the pressure of the free gas P_2 using (1.2) and (1.4) as follows:

$$q = \beta P_2 / (ZRT) \tag{1.6}$$

As the gas is filtered, the gas separation conditions undergo changes governed by the gas permeability coefficient k and diffusion coefficient. The latter are functions of the coordinates and time, the variation of porosity being described by [5]

$$m_i = m_i^0 \exp(\alpha_i,\ \sigma), \quad i = 2,\ 3 \tag{1.7}$$

Here m_2^0 and m_3^0 are the initial volume ratios of the macro- and micropores, respectively α_2 and α_3 are the compression coefficients, and σ is the hydrostatic pressure.

It has been established experimentally [6] that coal swelling depends strongly on the amount of gas, and has the form

$$e = KA \tag{1.8}$$

where e is the volume expansion of the gas-saturated cracked porous medium, A is the variation of the amount of gas per unit volume of the medium, and K is the swelling or volume expansion coefficient of the medium. We determine the change in the amount of gas as follows:

$$A = a - a_0 + (P_2 m_2 - P_{20} m_2^0) / (ZRT) \tag{1.9}$$

where a_0 and P_{20} are the amount of absorbed gas per unit volume of the medium and the free gas pressure inside the pores in an unperturbed rock mass or in any other initial state.

2. THE BASIC EQUATIONS OF THE MODEL

Following the main assumptions of the mechanics of heterogeneous media [7], to derive the equations of the stress–strain state of a non-linear gas-saturated cracked porous medium we will use the equation of continuity and the equation of motion for the kth component

$$\frac{\partial(\rho_k m_k)}{\partial t}+\frac{\partial}{\partial x_j}(\rho_k m_k V_j^k)=q_k \tag{2.1}$$

$$\rho_k m_k \frac{d_k V_i^k}{dt}-\frac{\partial}{\partial x_j}(m_k \sigma_{ij}^k)+F_i^k-V_i^k q_k=0, \quad i=1,\ 2,\ 3 \tag{2.2}$$

where x_j are the macrocoordinates used to set up the averaged equations, q_k is the mean mass transfer from the kth component into the other ones, m_k is the volume ratio, ρ_k is the density, V_j^k are the components of the mean velocity vector of the motion, and F_i^k are the components of the resulting force due to the momentum exchange between the kth phase and the other ones (inside the primary macrovolume). In (2.1) and (2.2) i and j ($i,\ j=1,\ 2,\ 3$) are the fixed and free indices, respectively, while σ_{ij}^k is defined in the same way as the mean phase stress on the surface.

Substituting the velocity determined from Darcy's law (1.1), the density from the equation of state (1.2), and the mean gas transfer (1.6) into the equation of continuity (2.1), we obtain the equation of motion

$$\frac{\partial}{\partial t}\left(\frac{m_2 P_2}{ZRT}\right)-\frac{\partial}{\partial x_j}\left(\frac{k}{\mu_1 ZRT}\frac{P_2\,\partial P_2}{\partial x_j}\right)+\frac{\partial}{\partial x_j}\left(\frac{m_2 P_2 V_j^1}{ZRT}\right)+\beta\left(\frac{P_2}{ZRT}-\frac{a}{k_0(a_0^0-a)}\right)=0 \tag{2.3}$$

for the gas inside the macropores and cracks.

By analogy, for the third component, substituting the expression for the velocity from Fick's law (1.3) into (2.1) and taking (1.6) into account, we obtain the equation of motion

$$\frac{\partial a}{\partial t}-\frac{\partial}{\partial x_j}D\left(\frac{\partial a}{\partial x_j}\right)+\frac{\partial}{\partial x_j}(aV_j^1)-\beta\left(\frac{P_2}{ZRT}-\frac{a}{k_0(a_0^0-a)}\right)=0 \tag{2.4}$$

for the gas inside the micropores.

To compute the stress state of the gas-saturated cracked porous medium in the vicinity of mining cavities and holes it is necessary to write the equations of the theory of elasticity that describe the process of medium deformation taking the influence of the gas into account. To this end we add the equations of motion (2.2) for all three components to get

$$\sum_{k=1}^{3}\rho_k m_k \frac{d_k V_i^k}{dt}-\frac{\partial \Gamma_{ij}}{\partial x_j}-q(V_i^2-V_i^3)=0 \tag{2.5}$$

Here it is taken into account that

$$F_i^1+F_i^2+F_i^3=0, \quad i=1,\ 2,\ 3 \tag{2.6}$$

and

$$\Gamma_{ij} = m_1\sigma^1_{ij} - (m_2P_2 + m_3P_3)\delta_{ij} \tag{2.7}$$

is the total stress tensor for a multicomponent medium, which can also be represented in the form

$$\Gamma_{ij} = G\left(\frac{2\nu}{1-2\nu}\delta_{ij}\,\mathrm{div}u + \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - N\delta_{ij}A \tag{2.8}$$

$$N = 2G(1+\nu)K/[3(1-2\nu)]$$

where u is the mean displacement vector of the matrix, ν is Poisson's ratio, G is the shear modulus of the medium, δ_{ij} is the Kronecker delta, and N is the stress factor in the medium due the presence of the gas. The gravitational force acting on the solid phase is neglected.

Substituting (1.1), (1.3) and (1.6) into (2.5), and taking (2.8) into account, we obtain the equations

$$\frac{\partial}{\partial x_j}\left[G\left(\frac{2\nu}{1-2\nu}\delta_{ij}\,\mathrm{div}\,u + \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)\right] =$$

$$= m_1\rho_1\frac{\partial^2 u_i}{\partial t^2} + N\frac{\partial A}{\partial x_i} + \beta\left(\frac{P_2}{ZRT} - \frac{a}{k_0(a_0^0 - a)}\right)\left(\frac{k}{\mu_1 m_2}\frac{\partial P}{\partial x_i} - \frac{D}{a}\frac{\partial a}{\partial x_i}\right),\ i = 1, 2, 3 \tag{2.9}$$

As was mentioned above, the gas permeability and diffusion coefficients depend on the stress state of the medium, while the elasticity characteristics depend on the gas saturation. This means that when studying dynamical processes one has to consider the above-mentioned physical quantities to be interrelated functions of the coordinates and time. Assuming all the other coefficients to be constant, one can write the filtration equations (2.3), the diffusion equations (2.4), and the equations of motion (2.9) of the medium as follows:

$$\frac{\partial(m_2P_2)}{\partial t} - \frac{k}{2\mu_1}\Delta P_2^2 - \frac{P_2}{\mu_1}\frac{\partial P_2}{\partial x_j}\frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j}\left(P_2m_2V_j^1) + \beta(P_2 - \frac{aZRT}{k_0(a_0^0 - a)}\right) = 0 \tag{2.10}$$

$$\frac{\partial a}{\partial t} - D\Delta a - \frac{\partial a}{\partial x_j}\frac{\partial D}{\partial x_j} + \frac{\partial(aV_j^1)}{\partial x} - \frac{\beta}{ZRT}\left(P_2 - \frac{aZRT}{k(a_0^0 - a)}\right) = 0 \tag{2.11}$$

$$(\lambda+\mu)\frac{\partial}{\partial x_i}\mathrm{div}\,u + \mu\Delta u_i - \frac{\partial\lambda}{\partial x_i}\mathrm{div}u + \frac{\partial\mu}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) = \tag{2.12}$$

$$= m_1\rho\frac{\partial^2 u_i}{\partial t^2} + N\frac{\partial A}{\partial x_i} + A\frac{\partial N}{\partial x_i} + \beta\left(\frac{P_2}{ZRT} - \frac{a}{k(a_0^0 - a)}\right)\left(\frac{k}{\mu_1 m_2}\frac{\partial P_2}{\partial x_j} - \frac{D}{a}\frac{\partial a}{\partial x_j}\right),\ i = 1,\ 2,\ 3$$

Here Δ is the Laplace operator and λ and μ are the Lamé coefficients.

Hence we obtain the system (2.10)–(2.12) of five differential equations consisting of the filtration equation for the free gas in the system of macropores and cracks, the diffusion equation for the absorbed gas inside the micropores, and the three equations of the theory of elasticity in terms of displacements. This system is the basis of the proposed mathematical model, with the aid of which we will study the dynamical state of a saturated cracked porous medium subject to vibration–wave interactions.

3. FORMULATION OF THE BOUNDARY-VALUE PROBLEM

To answer the question of how to choose the mathematical model it is necessary to state the boundary-value problem. In a rigorous mathematical formulation of the boundary-value problem it is obviously impossible to take into account all the details of the technological process involved in mining. We will make a number of simplifying assumptions to study the stress–strain state of a fixed seam in the vicinity of a hole drilled from within an excavation, at the surface of which there are no forces, while a vibrator is applied to the hole.

The general stress state can be represented as the sum of the stress state of the rock mass without an excavation and an additional stress state due to the presence of the excavation and the hole. Several authors have studied the stresses in unperturbed seams, i.e. the basic stress state. Theoretical work has shown that an unperturbed seam remains in a homogeneous stress state provided the seam is horizontal. In the case of an inclined seam the components of the stress tensor that occur in it are linear functions of the Cartesian coordinates defined by the following functional relationships

$$\sigma_{ij}^c = \sigma_{ij}(H,\ \gamma,\ \alpha_0, \beta_0,\ v_1,\ G_1,\ v_2,\ G_2,\ x,\ y),\ \ i,j\ =\ 1,\ 2,\ 3 \tag{3.1}$$

Here H is the distance between the stipulated centre of the seam and the free surface, γ is the mean density of the surrounding rock, β is the lateral outward pressure coefficient α_0 is the angle of inclination of the seam, and ν_1, G_1, ν_2, G_2 are Poisson's ratios and shear moduli of the rock and the coal seam, respectively.

We assume that prior to excavations the gas-saturated cracked porous rock seam remains in a static equilibrium state. Its macropores and cracks are filled with free gas under pressure $P_2 = P_0$, while the micropores contain absorbed gas with density $a = a_0$ per unit volume of the medium. These quantities are related by Langmuir's equation, which can be written in the form

$$a = a_0^0 k_0 P_0 \,/\, (ZRT + k_0 P_0) \tag{3.2}$$

It is taken into account that in an unperturbed rock mass remaining in the state of equilibrium the free gas pressure $\mathbf{P}_3$ in the micropores is equal to the gas pressure $\mathbf{P}_2$ in the macropores and cracks.

When analysing the stresses in the vicinity of an excavation one can assume the basic state to be homogeneous and defined by (3.1) in the domain under consideration, since the additional stress state has a local character in a domain comparable with the thickness of the seam, which is many times smaller than the limits within which the seam is extended at the depth H at which it is located. When the additional stress state is studied in the new local system of coordinates xyz the boundary conditions at the free excavation surface $x = 0$ can therefore be written in the form

$$\sigma_x = -\sigma_x^0,\ \ \tau_{xz} = -\tau_{xz}^0, \tau_{xy} = 0,\ \ P = P_c,\ \ a = a_c \tag{3.3}$$

where P_c and a_c are the pressure and sorption at the surface of the hole, respectively. Here and henceforth $m = m_2$ and $P = P_2$.

The study of the effect of an excavation upon the state of a saturated seam [8] revealed that the variation of stress in the rock mass, the free gas pressure distribution in the macropores and cracks, and the redistribution of the sorption in the micropores can be observed in a domain comparable with the thickness of the seam. At distances several times longer than the seam thickness the effect of an excavation can practically be neglected. This means that when studying the additional stresses in the neighbourhood of a hole in the seam drilled from within the excavation, one can write the boundary conditions far enough from the excavation surface in the form

$$\sigma_r = -\sigma_r^0 = -\sigma_z^0 \cos^2\varphi - \sigma_y^0, \ \sin^2\varphi, \ \tau_{r\varphi} = 0, \ \tau_{rx} = 0 \tag{3.4}$$

$$P = P_c, a = a_c$$

When studying the effect of a vibrator applied to the hole upon the coal seam we will assume that ideal contact is maintained between the vibrator and the surface of the hole. Suppose that radial displacements are generated at the surface of the vibrator, which can be described by the sinusoidal law

$$u = u^0 \sin\omega t \tag{3.5}$$

where u^0 is the amplitude of the generated displacement and ω is the vibration frequency. Then the boundary conditions of the problem of studying the additional stress–strain state of a gas-saturated coal layer in the vicinity of a hole vibrator will have the form (3.5), while the character of pressure and sorption variation at the surface must be defined as follows:

$$P = P_c(t), \ a = a_c(t) \tag{3.6}$$

Here $P_c(t)$ and $a_c(t)$ are the pressure and sorption boundary values, which in the general case depend on the time $t > 0$ and other technical characteristics of the vibrator.

The main goal of the present study is to state the boundary-value problem of mathematical physics concerned with the interaction of two related gas flows and the dynamical process of stress predistribution in a gas-saturated rock mass caused by the vibrator. Because of this it suffices to state the following ideal initial conditions.

Let the stress in the vicinity of the hole be described by (3.4) at the initial instant of time. We assume that the free gas pressure distribution and sorption inside the rock mass is homogeneous at the initial instant $t = 0$

$$P = P_0, \ a = a_0 \tag{3.7}$$

On the basis of (3.4) and (3.6) one can observe that in the general case the stress state in the vicinity of a hole cannot be considered to be axisymmetric. This would, however, lead to a very complex mathematical problem. In the first formulation we shall therefore make one more assumption, namely

$$\sigma_z^0 = \sigma_y^0 \tag{3.8}$$

which enables us to consider the free gas pressure, sorption, and stress fields to be axisymmetric in the vicinity of the hole.

The above assumptions therefore enables us to write the following equations in a cylindrical system of coordinates $r\varphi x$: the filtration equation for the free gas within the system of macropores and cracks

$$\frac{\partial(m\mathbf{P})}{\partial t} - \frac{k}{\mu_1}\left[P\frac{\partial^2 P}{\partial r^2} + \left(\frac{\partial \mathbf{P}}{\partial r}\right)^2 + \frac{P}{r}\frac{\partial P}{\partial r}\right]\frac{P}{\mu_1}\frac{\partial P}{\partial r}\frac{\partial k}{\partial r} + \\ + \frac{1}{r}\frac{\partial}{\partial r}\left(rPm\frac{\partial u}{\partial t}\right) + \beta\left(P - \frac{aZRT}{k_0(a_0^0 - a)}\right) = 0 \tag{3.9}$$

the diffusion equation for the absorbed gas within the system of macropores

$$\frac{\partial a}{\partial t} - D\left(\frac{\partial^2 a}{\partial r^2} + \frac{1}{r}\frac{\partial a}{\partial r}\right) - \frac{\partial a}{\partial r}\frac{\partial D}{\partial r} + ra\frac{\partial u}{\partial t} + \frac{\beta}{ZRT}\left(P - \frac{aZRT}{k_0(a_0^0 - a)}\right) \tag{3.10}$$

and the equations of motion of the gas-saturated cracked porous medium

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial r^2}+(2\lambda+3\mu)\frac{1}{r}\frac{\partial u}{\partial r}-(\lambda+\mu)\frac{u}{r^2}+\frac{\partial u}{\partial r}\frac{\partial(\lambda+\mu)}{\partial r}=$$
$$=m_1\rho_1\frac{\partial^2 u}{\partial t^2}+N\frac{\partial A}{\partial r}+\beta\left(\frac{P}{ZRT}-\frac{a}{k_0(a_0^0-a)}\right)\left(\frac{k}{\mu_1 m}\frac{\partial P}{\partial t}-\frac{D}{a}\frac{\partial a}{\partial r}\right) \tag{3.11}$$

The boundary conditions for the problem of studying the additional state due the action of the vibrator can be written in the form

$$P(r_0,\ t)=P_c(t),\ a(r_0,\ t)=a_c(t),\ 0<t<\infty \tag{3.12}$$

on the surface of the hole.

According to (3.7), the initial conditions can be written as

$$u(r,\ 0)=0,\ P(r,\ 0)=P_0,\ a(r,\ 0)=a_0,\ r_0<r \tag{3.13}$$

The boundary-value problem (3.9)–(3.13) consists of three non-linear second-order partial differential equations describing the dynamical state of a gas-saturated cracked porous medium. Numerical methods must be invoked to solve the system.

4. THE MODEL PROBLEM

Two problems, the connected and the disconnected one (neglecting the effect of matrix displacement on the gas flows), have been solved for low-frequency vibrations with constants $\omega=125.6\ s^{-1}$, $E=540$ MPa, and $\nu=0.35$. The remaining physical parameters have been chosen as follows: $m_2=0.02$, $\mu_1=1.2\times10^{-4}$ Pa s, $k=10^{-14}\ m^2$, $Z=0.18$, $D=10^{-5}\ m^2/s$, $\beta=3.5\times10^{-6}\ s^{-1}$, $K=0.24\times10^{-2}\ m^3/kg$, $a_0^0=24\ kg/m^3$, $k_0=0.35\ m^3/kg$, $R=529$ J/(kg K), and $T=298$ K.

Well-known approaches [8, 9] have been employed to construct the solutions.

Figure 1 shows graphs of the free gas pressure distribution (curves 1) in the vicinity of a hole of radius $R=0.05$ m at time $t=1$ s (the solid and dashed lines correspond to the disconnected and connected problems, respectively). It is assumed that the surface of the hole is under a periodic load due to the gas pressure $P_c=9.81\times10^4(1+0.2\sin\omega t)$ Pa.

Even though during this short time interval the pressure and sorption distributions in both problems are practically the same, the displacement distributions differ considerably (curves 2).

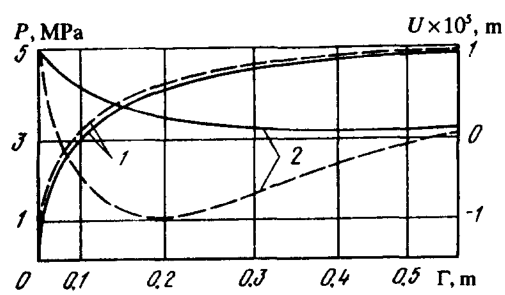

Fig. 1.

Therefore the solutions of the simplified model problems indicate that the computations based on the more-sophisticated mathematical model, which reflects the mechanism of the action of the vibration waves on the gas-saturated coal mass in the most complete way, enable us to determine the parameters of vibration treatment more accurately and also could reveal new vibration effects in the coal seam.

REFERENCES

1. POTURAYEV V. N. and MINEYEV S. P., *The Use of Vibration and Wave Effects in Bed Treatment in the Presence of Outburst Hazards*. Nauk. Dumka, Kiev, 1992.
2. PODIL'CHUK Yu. N., The theory of gas-saturated porous medium deformation. *Prikl. Mekh.* **12**, 12, 42–47, 1976.
3. PODIL'CHUK Yu. N. and LYAKH V. V., Investigation of the stress state of a gas-saturated rock mass near an ellipsoidal excavation. *Prikl. Mekh.* **16**, 9, 27–35, 1980.
4. SKOCHINSKII A. A., KHODOT V. V., GMOSHINSKII V. G. *et al.*, *Methane in Coal Seams*. Ugletekhizdat, Moscow, 1958.
5. KHODOT V. V., YANOVSKAYA M. F., PREMYSLER Yu. S. *et al.*, *Physical Chemistry of Gas Dynamics Phenomena in Mines*. Nauka, Moscow, 1973.
6. RYABCHENKO A. S., SEMENOV Yu. N. and SVETLANOV Yu. V., Swelling and the stress state of coal as a function of the gas-saturation. In *Combating Gas and Sudden Outbursts in Coal Mines*, pp. 53–60. Kemerovo, 1973.
7. NIGMATULIN R. N., *Principles of the Mechanics of Heterogeneous Media*. Nauka, Moscow, 1978.
8. LYAKH V. V., Investigation of the stress–strain state of a perturbed fixed bed. *Prikl. Mekh.* **19**, 11, 130–133, 1983.
9. LYAKH V. V., An approach to the analysis of the stress state of an inclined gas-saturated bed in the vicinity of an excavation. *Prikl. Mekh.* **25**, 9, 51–57, 1989.

Translated by T.J.Z.